Completing the square and turning point form

$\pm a(x \pm b)^2 \pm c$

The turning point form can be found either from a graph or factorising an expression by completing the square.

e.g. $y = x^2 - 4x + 6$

$y = (x-2)^2 - 4 + 6$

$\boxed{y = (x-2)^2 + 2}$

+ min t.p. $(2,2)$

$\cup$ graph

there are no factors of +6 that add to −4. so complete the square.

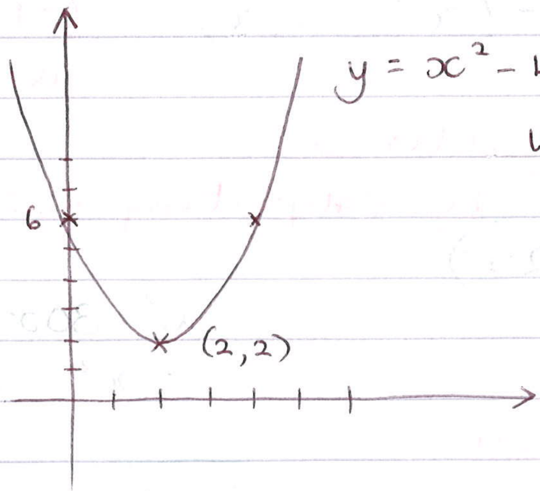
Remember

$(x-2)^2 = x^2 - 4x + 4$

+4 too much

so we must − 4

$(x-2)^2 - 4 = x^2 - 4x$



$y = x^2 - 4x + 6$

using

$y = \pm a(x \pm b)^2 \pm c$

$y = +a(x-2)^2 + 2$

use $(0,6)$

$6 = +a(0-2)^2 + 2$

$6 = +a(-2)^2 + 2$

$6 = 4a + 2$

$4 = 4a$

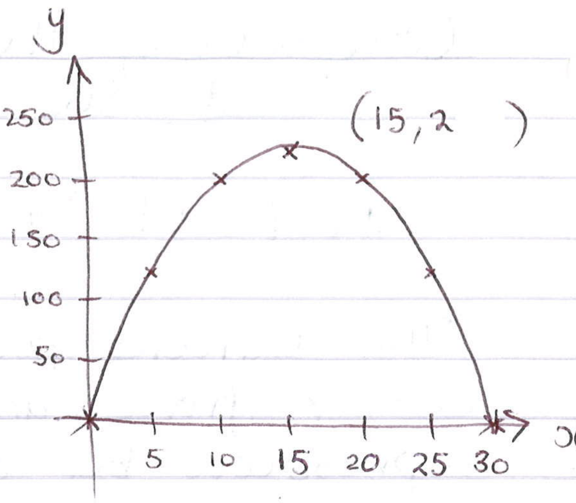
$a = 1$

the same!

so eqn of graph $\boxed{y = (x-2)^2 + 2}$

Another example

$y = 30x - x^2$



using graph to find t.p. form

$y = \pm a(x \pm b)^2 \pm c$

$y = -a(x-15)^2 + 2$

sub in $(0,0)$

$0 = -a(0-15)^2 + 225$

$0 = -a(-15)^2 + 225$

$0 = -a \times 225 + 225$

$0 = -225a + 225$

$225a = 225$

$a = 1$ $\quad y = -(x-15)^2 + 225$

or find t.p. form algebraically from $y = 30x - x^2$

$y = 30x - x^2 \quad = -x^2 + 30x$ (changed order)

$= -(x^2 - 30x)$ (-1 removed as common factor)

same

$y = -(x^2 - 30x)$ factorise by completing the square

$y = -((x-15)^2 - 225)$

$x -$

$= x^2 - 30x + 225$ too much

$x^2 - 30x = (x-15)^2 - 225$

$y = -(x-15)^2 + 225$

(multiplied both terms by -1)